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Optimal Boundary for Input Detection with LIF Neuronal Models*

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Abstract

We investigate information transmission in neuronal models based on Brownian motion and the Ornstein–Uhlenbeck process, in which neuronal spiking times are modeled as first passage times through oscillating boundaries. Using mutual information and mutual information per unit time as metrics, we analyze how boundary oscillation parameters and input variability influence coding efficiency. Our analysis reveals complex dependencies on input variability and diffusion strength, including non-monotonic effects of input variance and unexpected increases in information with diffusion strength in the Ornstein–Uhlenbeck case. We also identify the existence of optimal oscillation frequencies, whose values depend on the specific information measure used.

Keyphrases

Ornstein-Uhlenbeck process, first passage time, mutual information, input-specific information.

Introduction

Brownian motion, $\{B_t, t \geq 0\}$, and the Ornstein–Uhlenbeck process, $\{X_t, t \geq 0\}$, both characterized by the same diffusion coefficient σ^2 but different drift terms, μ and $\mu + \tau X_t$ (with $\tau > 0$), respectively, are widely used to describe the evolution of a neuron's membrane potential (Sacerdote and Giraudo 2012). These models, known as Integrate-and-Fire and Leaky Integrate-and-Fire models, represent neuronal spiking times as the first passage times (FPTs) of the underlying processes across a threshold. In this setting, the neuron acts as an integrator of external stimuli, with the input incorporated through the parameter μ. The FPT through a given boundary is thus interpreted as the neuron's output. A key problem in this framework is to infer unobserved input from observed output.

In this work, using the aforementioned models, we investigate how much information about the input parameter μ can be transmitted through the observed output. Specifically, we explore the existence of optimal boundaries, where optimality is defined in terms of maximizing information transmission. We focus on oscillating boundaries and examine both the existence of an optimal oscillation amplitude and the influence of oscillation frequency.

Methods

In our study, we treat the input μ as a random variable M with a given probability density function $f_M(\mu)$. The input-output relationship is then described by the conditional probability density function $f_{T|M}(t|\mu)$, representing the distribution of the first passage times $T = t$ for a given input $M = \mu$. The amount of information (in bits) about the input M conveyed by the output T is quantified by the mutual information $I(M;T)$ (Gallager 1968),

$$I(M;T) = \int_{\mathcal{M}} i(\mu;T) f_M(\mu) d\mu$$

where $i(\mu;T)$ is the input-specific information, which measures how much information is associated with particular input $M = \mu$ (Kostal and D'Onofrio 2017)

$$i(\mu;T) = \int_{\mathcal{T}} f_{T|M}(t|\mu) \log_2 \frac{f_{T|M}(t|\mu)}{f_T(t)} dt$$

and $f_T(t)$ is the marginal probability density of the output T

$$f_T(t) = \int_{\mathcal{M}} f_{T|M}(t|\mu) f_M(\mu) d\mu.$$

To enable comparison across different boundaries and distributions of the drift μ, we standardize our indexes. In addition to mutual information $I(M;T)$ and input-specific information $I(\mu;T)$ we also consider the mutual information per unit of time, denoted by $I_T(M;T)$. This quantity, which originates from the classical information-per-cost framework (McEliece 2002, Ch.2), is especially useful in our context, even though it is still relatively underutilized in computational neuroscience (Kostal and Lánský 2006). While mutual information $I(M;T)$ quantifies the number of bits that can theoretically be transmitted per input-output cycle, the output variable T represents time (i.e., FPT in seconds). Hence, it is natural to consider the information rate, or information per unit time (in bits per seconds), defined as:

$$I_T(M;T) = \frac{I(M;T)}{\langle T \rangle},$$

where $\langle T \rangle = \int_{\mathcal{T}} t f_T(t) dt$ is the mean FPT induced by the given probability distribution of input values.

For both the Integrate-and-Fire and Leaky Integrate-and-Fire models, we introduce an oscillating threshold defined by

$$S(t) = c + d \sin(2\pi a t)$$

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where c is the baseline threshold, d is the amplitude, and a is the frequency of oscillation. We assume that the random input M follows a shifted Gamma density with parameters $\alpha > 0$, $\beta > 0$ and shift $\mu_0 > 0$, given by

$$f_M(\mu) = \frac{\beta^\alpha}{\Gamma(\alpha)}(\mu - \mu_0)^{\alpha-1}e^{-\beta(\mu-\mu_0)}.$$

We allow both the mean and the variance of the input distribution Gamma distribution to vary. Our analysis focuses on how the mutual information $I(M;T)$ and the mutual information per time $I_T(M;T)$ respond to changes in the parameters of the boundary, particularly amplitude, d, and frequency, a, of oscillation.

Results

Some of the results obtained in this preliminary study are unexpected. For instance, when examining the dependence of mutual information on two types of variability—input variance Var(M) and diffusion strengthσ^2, a non-monotonic dependence emerges between mutual information and Var(M), even when the expected value $\langle M \rangle$ is held constant.

Furthermore, while increasing σ^2 generally leads to a decrease in mutual information $I(M;T)$ for both the Brownian Motion and the Ornstein-Uhlenbeck processes, in the Ornstein-Uhlenbeck case we observe counterintuitive scenarios where increasing σ^2 results in an increase in the mutual information per unit time $I_T(M;T)$.

Additional findings concern the existence of ane optimal oscillation frequency a. Interestingly, the optimal value depends on whether we consider total mutual information or mutual information per unit time. Moreover, in the Brownian motion model, the frequency that maximizes information is independent of σ^2, whereas in the Ornstein-Uhlenbeck model, it appears to vary with σ^2.

Conclusion

These results suggest that oscillatory mechanisms and stochastic variability can jointly enhance neural coding performance, providing nuanced picture than models with static thresholds. Future work should extend this analysis to more general input distributions, larger neuronal populations, and experimental validation, in order to better understand how oscillatory dynamics and noise contribute to information processing in the brain.

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