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The Essential Enquiry 'Equal or Equivalent Entities?' About Two Things as Same, Similar, Related, or Different*

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Abstract

We discuss definitions of entities, equality, and equivalence as used by a transdisciplinary diversity of research fields including mathematics, statistics, computational linguistics, computer programming, knowledge engineering, music theory, and genomics. Declaring definitions for these concepts in the situational context of each domain specific field supports the essential question 'Equal or equivalent entities?' about two things as same, similar, related, or different for that field. Pattern recognition performed by artificial intelligence applications whether with supervised and unsupervised learning or with deductive and inferential logic, with machine learning or logical reasoning, can be described as the automated process of answering this fundamental question about the similarity, relatedness, or difference between two things.

Keywords

DREAM principles, equivalent entities, semantic web, knowledge engineering, ontology, scientific reproducibility, plagiarism.

Contents

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Introduction

For as long as we've been on Earth, humans have used comparative analysis to make sense of the world around us. Identifying the distinct features between two objects, items, or persons is often how we define and recognize an entity. Regardless of the context, we use contrasts and comparisons to differentiate and establish an understanding of the existence of objects within our perceptions of reality. Perhaps, it is a matter of perspective within every individual being and of the context and situation in which an entity is viewed. In the pursuit of truth about the fundamental idea of reality, Greek philosophers grappled with comparisons to define what is considered 'good and evil'.

From the Enchiridion, St. Augustine [2] was most influenced by Neoplatonism and described the debate in the existence of evil within good and innate evilness within each entity, stating that "from this it follows that there is nothing to be called evil if there is nothing good". Furthering this philosophical argument, Augustine wrote a book titled *City of God* stating, "For evil has no positive nature; but the loss of good has received the name evil". This question directly results from the necessity to compare the existence between evil and good where we cannot call anything evil if there is nothing good to compare it to, and vice versa [3]–[6]. From a philosophical perspective, when defining a term, idea, or effect as 'good' it implies the existence of an opposite effect: an evil. This concept has taken the form of the privation theory of evil, otherwise known as the 'absence of good' in which evil is considered to be a virtue that is lacking in a specific entity, and it is incorrect to define evil as a substantiated entity [7]. A specific comparison often used by Aristotle to identify the term 'privation' would be blindness, for which blindness is bad because it is a privation of sight; not simply an absence, but rather an absence of a quality from an entity [8]. In fact, numerous Greek philosophers employed an "argument by analogy" [9] as a means of justification of similarity between two entities as proof that they are true of each other; while this may not be always effective evidence, it indicates our dependence on contrasts and comparisons in defining entity value [10]. In evaluating contrasts and comparisons, we seek to find the relations and/or distinguishing features between two entities, that will answer the fundamental question of 'Equal or Equivalent Entities?'

In this paper, we discuss the question of 'Equal or Equivalent Entities?' and the citation of equivalent entities to pursue fair citation, truth, and scientific progress. More specifically, we will discuss 'When are two entities equal?', 'When are two entities equivalent?', 'How do we know whether two entities are equal, equivalent, or neither?', and

'What is the difference between equality and equivalence?'. Then, we will discuss equivalence in the context of several distinct problem domains and finally discuss the implementation of the terms same, similar, related, and different in database management systems as well as ontology and knowledge engineering. *Scientia nihil aliud est quam veritatis imago*: science is but an image of the truth (Francis Bacon, former Lord Chancellor of England) [11]. If we continue to pursue scientific enquiries as a means of scientific progress, it is essential that we evaluate the fundamental concept of scientific reproducibility with the citation of equivalent entities.

Contrasts and Comparisons in Context

As a species, humans have evolved to understand the world around us by adapting for survival and learning to differentiate friend from foe. But how do we discern when two things are same, similar, related, or different? Contrasts distinguish one thing from another with emphasis on the differences, while comparisons examine analogies, similarities, or relative rankings of related or similar things. The concept of contrasts and comparisons in contexts expand into everyday applications of natural phenomena. It is primarily demonstrated through our knowledge of the five senses: sight, sound, smell, taste, and touch[12]. A visual comparison for sight can be between the camouflage of an iguana or a chameleon who change color to blend into their environments. Another instance could even be the comparison of doppelgängers, identical and fraternal twins, and whether they could be considered similar or related. A comparison of sound can be heard in the related notes for music notation and composition. As humans, our smell perception is extremely sensitive in distinguishing between odors and determining what is 'good or bad' as a smell. In cooking, recipes are often built on existing inspiration for taste. Haptic memory is a type of sensory memory based on touch stimuli for interacting with familiar objects [13]. General concepts about contrasts and comparisons can be applied to analytic methods for diverse scientific research fields. In biology, comparisons between two or more creatures enable their classifications into the different species, genus, family, *etc.*, of the taxonomic hierarchy. In mathematics, comparisons between two or more numbers, or two or more functions over a domain with values in a range, enable their analysis with the formal definitions for basic operations such as $\min(\cdot), <, \leq, =, \geq, >, \max(\cdot)$, *etc.* In statistics, formal methods exist with hypothesis testing for the null versus alternative hypotheses when calculating the differences between two sets of data. In knowledge engineering, words listed in different vocabularies, thesauri, and ontologies representing concepts and ideas can be mapped to each other as same, similar, related, or different, and then applied to the review of published literature. This analysis remains an important requirement for integrity and ethics in scholarly research to evaluate novelty versus plagiarism [14].

Equal or Equivalent Entities?

Prior to considering equal or equivalent, we must consider what is the definition of an entity. As a method of review for the definitions of entity, equal, and equivalent, we identified three dictionaries(Merriam-Webster Dictionary, Oxford English Dictionary, Cambridge Dictionary) to evaluate the comparable and contrastable attributes between these definitions. From the Merriam-Webster dictionary definition of entity [15], an entity is "the existence of a thing as contrasted with its attributes". Defined by the Oxford English Dictionary[16], the definition of an entity is "A thing with distinct and independent existence". Cambridge Dictionary [17] defined an entity as "something that exists apart from other things, having its own independent existence".

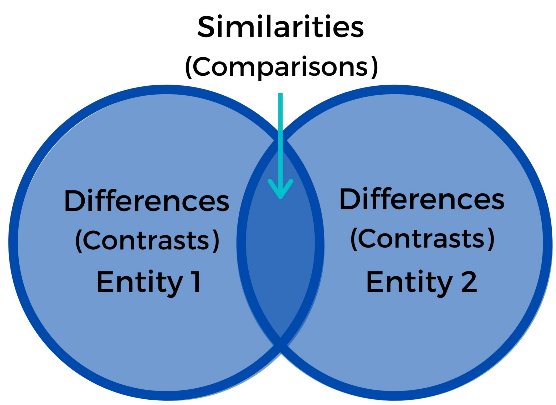

Figure 1: Entity Contrasts and Comparisons

Based on this definition of entities, we can consider the usage of equals versus equivalent. Contextually defined within Merriam-Webster Dictionary, *equal* is defined as "like in quality, nature, or status" and *equivalent* is defined as "corresponding or virtually identical especially in effect or function, like in significance or import". Oxford English Dictionary defines *equal* as "Being the same in quantity, size, degree, or value" and *equivalent* as "Having the same or a similar effect as another thing". Cambridge Dictionary defines *equal* as "the same in amount, number, or size" and 'equivalent' as "something that has the same amount, value, purpose, qualities, etc. as something else". Dictionary definitions regarding *equivalent* and *equal* implicitly assume the existence of context between two entities, but we explicitly define that *equal* and *equivalent* need to be defined in context by choosing what attributes need to be compared. The definitions of *equal* and *equivalent* can be applied in the comparison of two entities in an identifiable set of comparable attributes within the same problem domain. The significant differences between the dictionary definitions of the terms entity, equal, and equivalent demonstrate the necessity to identify our own definitions within the PORTAL-DOORS Project.

In the context of the PORTAL-DOORS Project(PDP) for the semantic web, an *entity* is defined as "the object of interest considered by the registrant to be the resource whether concrete or abstract, online or offline, semantic or lexical, real or virtual" [18]. Two entities are considered *equivalent* with meaning when there exists sufficient semantic similarity in the effect, purpose, meaning, and significance of an entity, despite lexical differences, including differences in natural language (*i.e.*, English, French, Russian, *etc.*). However, entities are defined as *equal* when each field in the metadata record is the same lexically and semantically, or considered the same value or quantity. Contrasts and comparisons are defined between six terms: equal, equivalent, same, similar, related, and different. NPDS records can be considered 'entities' within PORTAL-DOORS in which we intentionally create these software artifacts with meaning. The definition of an entity acknowledges the possible difference in meaning or intention by the registrant for the creation of "an object of interest" to be entered as a metadata record. To what extent do we consider an entity to have "an attribute"? When asking whether two entities are equivalent, you need to define which features you are comparing and to what degree of observation these features can be determined at. As such, you are making a choice in what attributes need to have equal values for two entities to be considered equivalent. The use of 'equal' and 'equivalent' can be determined by identifying a threshold of similarity [19].

The term 'equivalent entities' and the question 'Equal or equivalent entities?' both represent the essential enquiry of identifying and characterizing two entities as the same, similar, related, or different from each other. The PORTAL-DOORS Project (PDP) articulated a collection of design principles in 2006 for the continuing development of the Nexus-PORTAL-DOORS-Scribe (NPDS) cyberinfrastructure. The PDP design principles were renamed in 2019 as the DREAM principles with the phrase *Discoverable Data with Reproducible Results for Equivalent Entities with Accessible Attributes and Manageable Metadata* in support of scientific reproducibility and integrity in scholarly research [14]. A question about equivalent entities asks "What is same?" vs. "What is similar?" vs. "What is related?" vs. "What is different?" The importance of equivalent entities can be utilized in the preservation of fair citation in scholarly research and academia. Quoting from Dutta *et al.* [20],

> We emphasize that science will be neither reproducible nor fair without recognition, acknowledgment, attribution and citation of equivalent entities regardless of whether those equivalent entities are considered to be scientific hypotheses, scientific experiments, scientific data, scientific results or published articles in the scientific literature.

We examine this question about equivalent entities in a variety of fields including mathematics, statistics, computational linguistics, computer programming, music theory, genomics, and ontology engineering.

Mathematics: In mathematics, logical and numerical entities are defined formally within a context of axioms, postulates, and theorems, which provide the basis for more complicated relations, calculations, and analyses. Most commonly, equality is denoted by the equal sign '=' to indicate two values as being the same on either side of the equation. Outside of the basic definition of equals (=), the definition for equivalence is typically defined within the context of application though often is based on the original concepts of similarity and congruence.

Euclid, also referred to as the father of geometry, defined five central axioms for equality: (1) things which are equal to the same thing are also equal to one another, (2) if equals be added to equals, the wholes are equal, (3) if equals be subtracted from equals, the remainders are equal, (4) things which coincide with one another are equal to one another, and (5) the whole is greater than the part [21]. In set theory, {A, B, C} and {1, 2, 3} are equivalent sets because they contain three units, therefore indicating that equivalence depends on the choice of identification [22]. Within an equivalence class, elements are added if and only if they are equivalent. If A B, then A has equivalence with respect to B in the equivalence relation [23].

Defining an equivalence adheres to the necessity of consolidating the amount of examinable properties between two entities or sets including length, value, and degree, deeming them as similar and having the same effect or meaning, rather than equal, which considers two entities to be the same, semantically and lexically [24]. A congruence relation is a particular type of equivalence relation when two mathematical objects are equivalent such that any algebraic function applied to either will yield an equivalent result, especially within geometry [25].

Various definitions for equality and equivalence exist throughout mathematics and their applications from basic arithmetic to sophisticated computational algorithms.

Computer Programming: In programming languages such as C#, the equality operator '==' for reference types compares the reference identities and indicates that both the type and value are the same and checks that both pointers point to the same data in memory. The '.Equals()' method is a virtual method that can be overriden and thus mapped to define an equivalence operator for the type and value of the class-defined object [26]. This method ensures for a given class, one gets to choose which attributes of two instances in the class to compare to determine whether or not they are equivalent.

Truth tables are another concept in mathematics-based logic where statements are compared and contrasted. Typically, the statements are denoted by propositional variables, where they can be classified as either true or false. Expressions that can be compared with operators include: $\neg$ ("not"), $\vee$ ("or"), $\wedge$ ("and"), $\rightarrow$ ("conditional"), or $\leftrightarrow$ ("biconditional"). The comparing expressions can be described as a conjunction, where both statements are true, a disjunction, when either one or both of the statements are true. These tables can assess whether expressions are logically equivalent or not and test the validity of the structure of arguments or statements [27]. This concept has applications in computer programming and boolean algebra to identify logical equivalence.

Biomedical Statistics: Within statistics, equality and equivalence are attributed to several instances of comparison within what is known as the equivalence test, or t-test. The term equivalence means that the efficacies of two types of studies, the 'entities', are close enough to be considered equivalent. In biomedical statistics, equivalence can be tested with a 'margin' denoted by δ, The equivalence margin, denoted by δ, is part of an equivalence test, relatively common in biostatistics, where two treatments are tested for similar effects within a range of values specified by the margin δ [28]. The TOST equivalence test[29], or the two one-sided test method can be used to equivalence defined by margin with a lower and upper bound. When a null effect is reached, a researcher must be able to provide support for that hypothesis. Equivalence tests, such as the TOST test, indicates the presence of a more extreme effect and seeks to reject the null hypothesis instead of claiming the absence of an effect [30].

Computational Linguistics: While axioms and tests define equivalence in mathematics and statistics, computational linguistics necessitate observing the syntax behind languages and computer models behind natural human processes. In computational linguistics, an equivalence studied for natural language processing examines whether two languages use the same set of abstract syntax [31]. The concept of equivalence depends to a large extent on the definitions of semantics and meaning within a given model of language. One must identify the type of translated text and the function of the message itself. Translational models of equivalence should account for several variations of syntax difference of text, but the meaning must remain the same.

Equivalence translation entirely necessitates establishing the semantic equivalence of the source language text as quoted from Lewandowska-Tomaszczyk, "equivalence practice depends on the

Figure 2: Instances of 'Equal or Equivalent Entities?' Evaluations with Same, Similar, Related, or Different

type of text translated (e.g., translation of a media or legal text requires a different approach than the translation of a poetic form) and the function of the message (e.g. film translation requires fulfilling a number of technical conditions and constraints, absent in the translation of fiction" [32]).

Music Theory: Equivalence classes are equivalences between unordered sets and twelve-tone rows in ordered sets. Transpositional equivalence is a movement of the key center in an ordered set or a set of notes, primarily within the same tone system in a constant interval. Equivalence by transposition is the concept that chords mapped onto another are considered same or similar when transposed. Enharmonic equivalence is established between two notes, intervals, or scales that sound the same as one another, but are notated differently. This is depicted through the comparison of F♯ and G♭ which are the same physical key, but are necessary to differentiate when cycling through different scales [33]. Octave equivalence is when musical notes are separated by one or more octaves separated by frequency. As such, the two notes are notated the same, indicating those two notes as 'equivalent'. For example, within a Western diatonic scale, the note exactly an octave above B is B. Hence, all notes containing B♭ within a specific interval in an octave would be considered as part of the same pitch class. Therefore, the notes are same in notation but similar in range for the defined pitch class [34].

Genomics: Genomic equivalence is defined as the theory that all cells of an organism contain an equivalent complement of genetic information in the field of developmental biology. Primarily discovered in the regeneration of excised tissues, the presence of identical genes within somatic cells that are otherwise considered different from each other is still considered as 'genomic equivalence'. This presents a paradox in developmental biology that is only solved when considering differential gene expression. Within amphibians, such as the salamander, few obtain the ability to regenerate cells from the iris. The formation of the lens from the produced differentiated cells is considered a form of metaplasia [35], indicating that each of the cells regenerated share the same genes during the gene differentiation process. Indicating the presence of same genes within different cells thus forms an equivalence between the two cells that share similar core information within its genome. Another example is the process of synonymous mutation where the nucleotide sequence in the DNA changes, but the produced protein amino acid sequence does not. The change is con-

Table 1: Explanation of Coded Instances in Figure 2

Code	Associated Explanation
A1	Sharing all comparable attributes, not different
A2	Some characters different, same problem domain
A3	Same lexical etymology, same problem domain
A4	Unlike lexical syntax, same problem domain
B1	Some comparable meanings, same syntax
B2	Some different syntax and meanings, same problem domain
B3	Same lexical etymology, same problem domain
B4	Unlike lexical syntax, two or more different meanings
C1	Same lexical syntax, associated problem domain
C2	Some syntax different, associated problem domain
C3	Similar lexical etymology, associated problem domain
C4	Unlike lexical syntax, associated problem domain/meaning
D1	Same lexical syntax, distinct problem domains
D2	Some syntax difference, distinct problem domains
D3	Same lexical etymology, distinct problem domain
D4	Unlike lexical syntax, distinct problem domains

Table 2: Properties of Lexically and Semantically Same, Similar, Related, or Different

	Equal Entities	Equivalent Entities	Similar Syntax	Same Domain	Comparable Attributes	Shares Characters
Lexically Same	True	True	True	True	True	True
Semantically Same	True	True	True	True	True	True
Lexically Similar	False	False	True	True	True	True
Semantically Similar	False	True	False	True	True	True
Lexically Related	False	False	False	True	False	True
Semantically Related	False	True	False	True	True	False
Lexically Different	False	False	False	False	False	False
Semantically Different	False	False	False	False	False	False

sidered an 'evolutionary substitution' at the third base of codons for another codon for the same protein [36].

True or False Equivalence

In the examples above as well as those from other diverse fields of science, when the question 'Equal or equivalent entities?' has been asked and answered with uncertainty as an equivalence, then the corollary question 'True or false equivalence?' must also be considered. If either the data about the two entities are incorrect or the analysis algorithm is flawed and invalid, then the result of equivalence must be considered a false equivalence rather than a true equivalence [37]. While the leveraging of this data occurs in every field where the question of "Equal or equivalent?" is introduced, when two items are not necessarily supposed to be equal but are still considered true, a false equivalence occurs, which is when two compared things are deemed as equal based on inaccurate or false reasoning. When reviewing results from automated artificial intelligence applications, human experts should apply both logical reason and intuitive common sense when deciding between a true equivalence or a false equivalence.

Same, Similar, Related, or Different?

Defining same, similar, related, and different in order to determine equality or equivalence of entities must be declared in the context of a problem-oriented domain for a specific field of scientific research. Between two entities, we evaluate it between semantic and lexical analysis to compare and contrasts the selected attributes. Lexical comparison processes two entities as character strings without embodying its semantic characteristics [38]. Semantic comparison processes two entities with reference to word meaning in the context of defined vocabularies, thesauri, or ontologies [39].

Figure 2 presents pairs of pictographs in the cells of a $4x4$ table with 16 cases demonstrating simple examples for contrast and comparison of lexically and semantically same, similar, related, and different. The definitions associated with each individual use-case are explicitly stated in Table 1 which provides a set of 16 definitions that correspond to the 16 cases depicting examples of same, similar, related, and different. Cell B3 indicates the comparison of emigrate and immigrate as lexically related and semantically similar. Emigrate and Immigrate both share the word portion 'migrate' which according to Merriam-Webster Dictionary, is defined as "to move from one country, place, or locality to another". They are considered lexically related because they originate from the same etymology of *migrare*, and semantically similar as they both mean to move from one country to another.

Table 3: Mapping between Terms in PDP and OWL

Term	PORTAL-DOORS Project (PDP)	OWL Ontology
Entity	the object of interest considered by the registrant to be the resource	a "thing" described in a document
Equal	same value or amount in meaning and syntax	*Same Individual* axiom
Equivalent	same or similar in object, effect, purpose, significance	*owl:EquivalentClasses(:Person :Human)*
Same	sharing all comparable attributes	*owl:sameAs*
Similar	sharing comparable attributes with more similarities than differences	set of similar individuals in a class
Related	associated problem domain, same etymology	——
Different	unlike meaning and syntax	*owl:differentFrom*, *owl:AllDifferent*, *owl:DifferentIndividuals*

However, emigrate is defined as "to leave one's place of residence or country to live elsewhere" and immigrate is "to come into a country of which one is not a native for permanent residence" [15]. Therefore the slight differences in their compared definitions indicate that 'emigrate' and 'immigrate' and semantically similar, but lexically related.

Identifying equality and equivalence begins with the comparison and contrast of a set of attributes between two entities within four terms: same, similar, related, different. We explicitly define these terms in conjunction with each other for use of database management in the PORTAL-DOORS Project.

Same describes sharing all comparable attributes in meaning and/or lexical syntax and can be considered equal and equivalent. The use of 'same' does not necessitate equality and may be considered solely equivalent in certain instances.

Similar means of an alike type, where two entities have two or more attributes that are not entirely the same in meaning or syntax(in a threshold of similarity) and can be considered equivalent. This may or may not be equivalent and equivalence is not required to be similar.

Related things that may be semantically related within associated problem domains and/or lexically related with the same etymology(lexical word origin) despite lexical and semantic differences, not considered equal or equivalent.

Different distinguishes two entities as within distinct problem domains as semantically different with unlike meanings and/or lexically different with unlike lexical syntax, not considered equal or equivalent.

Same, Similar, Related, and Different can be mapped onto the terms 'Equal' and 'Equivalent' in determining the comparison or contrasts between two entities in regards to the enquiry of "Equal or Equivalent Entities?" The definition of 'similar' also considers a certain threshold of similarity between several attributes of two entities rather than a binary question. This context imbues meaning and sense for the hierarchies and associations found within domain-specific ontologies not only for entities but also for the relationships between entities [40]. Table 2 describes the overlap between the definitions in each use case and the association of semantically or lexically same, similar, related, or different as corresponding within a specific characteristic. Then equivalence between concepts can be established for both entities and relationships.

The definitions of "same, similar, related, and different" contextualized within the semantic web are representative of the different variations of relationships between entities. Therefore, considering the conjunction 'AND' with same and similar and related and different in existing literature of the semantic web, excluding any one of these key terms does not present the whole picture in the comparisons and contrasts between two entities. Mapping the terms 'entity', 'equal', and 'equivalent' onto each of the definitions of same, similar, related, and different with semantic and lexical analysis remains at the core of the question "Equal or Equivalent Entities?". Exclusion of the the term 'related' or 'similar' in these set of essential terms causes uncertainty in use-cases that distinguish two entities as 'related' with the same lexical etymology or semantic attributes or 'similar' with comparable attributes, but not entirely equal.

Ontology matching and merging techniques are used to observe concept similarity between all types of relationships between the lexical and semantic concept and relationship name. Ontology matching techniques observe the relationship determination between elements of different ontologies. When their elements align, it defines the connections between concepts [41]. Also commonly used, individual concept-mapping is an approach to capture and present semantic knowledge and its conceptual organization for ontologies as well as display the similarity between two entities of comparison within a domain ontology. Described by Keet [42], foundational or overarching ontologies are commonly used to assist ontology development by providing a base structure to expand upon. However, although knowledge representation and concept-mapping via ontologies allows well-defined semantics for hierarchical relationships, associative relationships do not and therefore, must consider the semantic relationship between two compared entities.

In music theory as a dramatic example of the importance of situational context, a pitch is heard in relation to the other pitches around it. Even if using the same set of notes, by changing the starting position in the sequence of notes, the perceived mode and sound changes. Thus, musical scales can be similar and related, but not the same, despite having the same set of notes [43]. Related sets of tones may have similar pitch values, but for the definition of similarity within music, they are only related due to the difference in sound based on the context. In general, comparisons between two entities for equality or equivalence can be made by observing for the presence of common shared attributes between the two entities using similarity measures to characterize identifiable attributes [44]. The greater the number of similar attributes the two entities share in common, the greater the overall similarity between the two entities. Few knowledge representation systems within the semantic web identified that there exists a commonality in the definitions of any common concept. If two entities share a 'same' or 'similar' definition, the reasoning agent would allow those two entities to be retrospectively linked on the semantic web [45]. Within an OWL ontology class, the built-in OWL property *owl:sameAs* is used to link an individual entity to another individual entity, indicating that the two URI references refer to entities that encode a single attribute on a scale of similarity [27], [46], [47]. These properties are often used in concept mapping and linking between two entities, but maintain their difference in name. The *owl:differentFrom* property determines that the URI references of two entities refer to different individuals [48].

In distributing data across the semantic web, an entity can share information across several servers. Servers are equipped to contain the description of a single entity as well as similar properties; any server should be able to specify any property of an entity about which it contains information such as the URI reference [49]. Within linked data, equivalence classes indicate a connection formed between two properties that ensure the same meaning, but are not notated the same; using *rdfs:subPropertyOf* can make one property equivalent to another and define levels of related properties between two entities [49]. Especially in database management systems, flagging equal and equivalent records using shared properties is considered when merging records. However, in mapping equivalent sets, identical classes would be mapped onto each other; any set of *rdfs:subClassOf* triples would determine equivalence between the two classes. In federated data management, two URI references can be equated to refer to the same entity; when two classes are equivalent, they share meaning in similar members of the classes, but each class maintains its individual *rdfs:label* [49]. Equivalent classes in the OWL ontology can be defined using the *owl:equivalentClass* property which relate to classes of objects and their similar properties rather than individuals. The OWL 2 ontology contains several subsets of language OWL where structures found in one subset may not always translate to the others. The two languages can be defined as similar, related, or different depending on the threshold of contrastable terms; however, they may still contain the same constructs and meanings within the language [49].

Table 3 describes the comparison between the defined PDP definitions for entity, equal, equivalent, same, similar, related, different, and their comparable representations within the OWL ontology. Given a declared minimum threshold for the number of shared attributes required for similarity, the two entities can be considered similar or different, respectively, when the number observed is above or below the threshold [50]. If all of the observed attributes are similar for the two entities, then certainly they are equivalent. But whether they are also equal depends on the context of the domain specific question. The definitions for equal, equivalent, same, similar, related, and different will be embedded within the PDP-DREAM ontology to describe the principles about merging and flagging records in the PORTAL-DOORS Project and NPDS. Equal entities are considered the same lexically and semantically in all fields of the metadata record and will be merged while equivalent entities will be flagged as same or similar depending on the degree of similarity in their defined comparable attributes.

Conclusion

Our everyday tendency to compare and contrast things should also be present in our scientific research as a means of assessing both shared and unshared attributes between two entities. In the contrasts and comparisons between two entities, there should exist collections of attributes for which we can ask the question 'Equal or Equivalent Entities?' and evaluate whether the attributes are same, similar, related, or different. The fundamental concepts identified in this report elaborated how the terms *entity, equal, and equivalent* map to definitions for the terms *same, similar, related, and different* as well as their use in associative relationships between entities within the context of domain-specific fields and problem-oriented domains. Moreover, we examined this essential enquiry and interpretations of equality and equivalence within seven different fields: mathematics, statistics, computational linguistics, computer programming, music theory, genomics, and knowledge engineering. We discussed concerns about true equivalences and false equivalences within artificial intelligence applications, and described the definitions of same, similar, related, and different in applications involving database management systems when merging equal records and flagging equivalent records in the PORTAL-DOORS Project. The citation of equivalent entities remains essential to uphold and maintain scientific reproducibility and research integrity. The definitions within this paper of entity, equal, equivalent, same, similar, related, and different are implemented within the PDP-DREAM ontology for equal or equivalent entities. In understanding the transdisciplinary approaches in which we can view, cite, and comprehend the scientific literature, we can also expand the applications of the essential enquiry 'Equal or Equivalent Entities?' to real-world databases and maintain the integrity of our artificial intelligence applications when identifying equivalent entities.

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